Math 101 Benchmark 1

September 22, 2022

Name: _____ Student ID: _____

Indicate your section/instructor.

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Answer the questions in the spaces provided on the question sheets. Show an appropriate amount of work (including appropriate explanation) for each problem, so that graders can see not only your answer but also how you obtained it. Include units in your answer when possible. You may receive 0 points for a problem where you show no work.

Instructions:

1. Do not open this exam until you are told to do so.

2. Write your initials on every page!

3. No books or notes may be used on the exam.

4. You may only use an *approved* calculator on the exam. If you have a problem with your calculator, raise your hand.

5. Read and follow directions carefully.

6. All cell phones must be turned off and put away during the exam. Any device that connects to a phone or the web must be removed and put away.

7. Do not separate the pages of this exam. If they do become separated, point this out to your instructor when you hand in the exam.

8. Make sure your answer is clearly marked.

9. Credit or partial credit will be given only when the appropriate explanation and/or work is shown.

10. This exam has 7 questions, for a total of 60 points. There are 7 pages besides this one.

11. You will have 90 minutes to complete the exam.

12. If you use graphs or tables to find an answer, be sure to include an explanation and sketch of the graph, and to write out the entries of the table that you use.

You can use this page for scratch work.

1. Answer the following questions about the table below:

x	-1	0	1	2	3	4
h(x)	-1	0	-1	3	-6	15

(a) (3 points) Does the table represent a function? **Explain** your answer in a sentence or two.

(b) (5 points) Fill in the following blanks with the indicated values:

$h(4) =$ [ ] $h(h(2)) =$ [ ] $h^{-1}(3) =$ [ ]

(c) (2 points) When does $h(x) = 15$?

$x =$ [ ]

2. Below is a graph of a function $f(x)$:
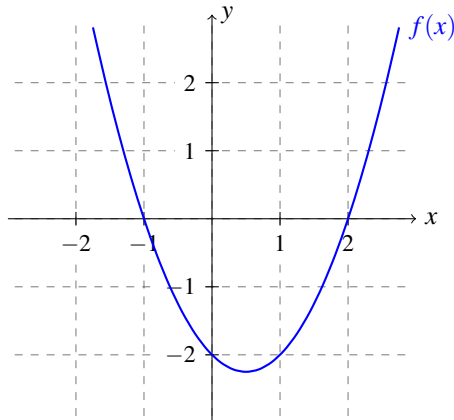
(a) (2 points) **What are $f(0)$ and $f(2)$?**

$f(0) =$ [ ]

$f(2) =$ [ ]

(b) (4 points) Compute the average rate of change of $f(x)$ on the interval $x = 0$ to $x = 2$. Write your **final answer** in the box below, and be sure to **show all work** to receive credit.

Average Rate of Change $=$ [ ]

3. Suppose that the thickness of the ice on a given lake (in inches) after t^{th} day of April is given by

$$H(t) = -0.2t + 6.5$$

(a) (3 points) What is the thickness of the ice on April 10th? Write your **final answer** in a **complete sentence** including **units**, and be sure to **show all work** to receive credit.

(b) (3 points) If the minimum thickness needed to safely ice fish on the lake is 4 inches, will it be safe to fish on April 20th? Write your **final answer** in a **complete sentence** including **units**, and be sure to **justify your answer** to receive credit.

(c) (3 points) If the minimum thickness needed to safely ice fish on the lake is 4 inches, what is the last safe day to fish on the lake? Write your **final answer** in a **complete sentence** including **units**, and be sure to **show all work** to receive credit.

4. This problem involves equations of lines. Be sure to **show all work** on all parts to receive credit.

   (a) (2 points) Find the equation of line A, which passes through the points $(1,2)$ and $(6,-3)$.

   (b) (2 points) Find the equation of line B, which has slope $\frac{1}{3}$ and passes through the point $(3,-1)$.

   (c) (2 points) Which line has a greater y-intercept, line A or line B? **Explain** your answer in a sentence or two.

   (d) (3 points) Where do the lines $y = -x + 1$ and $y = \frac{1}{2}x - 2$ intersect? Give the (x,y)-coordinates of the point where these lines intersect. Write your **final answer** in the box below, and be sure to **show all work** to receive credit.

   Point of Intersection $=$ ⬚

5. A paper airplane is thrown off a roof and glides to the ground. Suppose that the height of the paper airplane (in feet above the ground) is given by the function $h(t) = -\frac{7}{2}t + 28$, where t is the number of seconds after the airplane was thrown.

   (a) (3 points) Find and interpret $h(3)$. Write your **final answer** in a **complete sentence** including **units**, and be sure to **show all work** to receive credit.

   (b) (3 points) How tall is the building that the plane is thrown from and how do you know this? Write your **final answer** in a **complete sentence** including **units**, and be sure to **show all work** to receive credit.

   (c) (3 points) What values of t will accurately describe the height of the paper airplane? That is, what would be the **domain** of the function $h(t)$ for which the height of the airplane is accurately described? **Explain** your answer in a sentence or two, and be sure to **show all work** to receive credit.

6. Below is a graph of a function $f(x)$ with some (x, y)-coordinate points labeled:
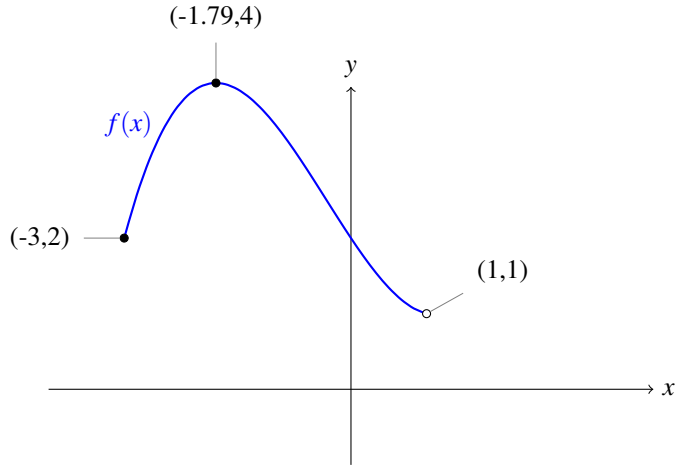
(a) (2 points) What is the domain of $f(x)$?

Domain of $f(x)$ = [   ]

(b) (2 points) What is the range of $f(x)$?
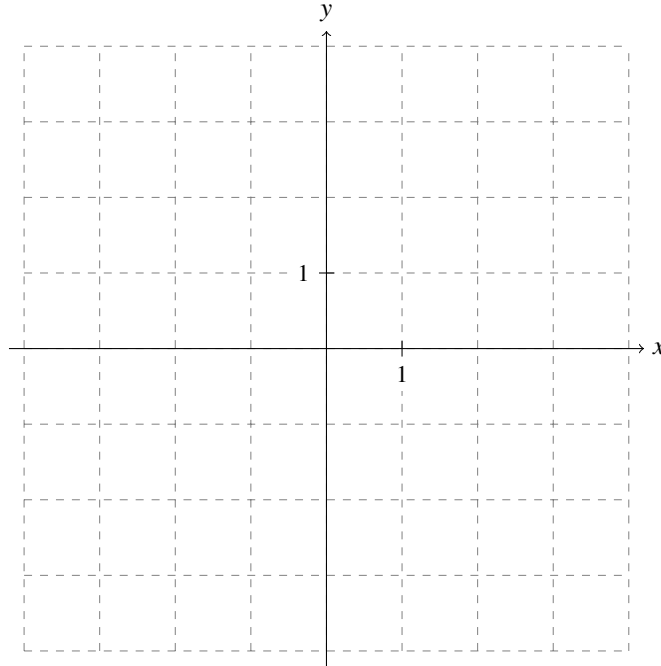
Range of $f(x)$ = [   ]

(c) (3 points) Now consider a different function $g(x) = \dfrac{1}{x^2 - 4}$. What is the domain of $g(x)$?

Domain of $g(x)$ = [   ]

7. Consider the function

$$f(x) = \begin{cases} x+1 & x < -1 \\ -2x+2 & -1 \le x < 2 \\ 1 & x \ge 2 \end{cases}$$

(a) (4 points) **Draw a graph of the function $f(x)$ on the grid above.**

(b) (2 points) Find $f(-2)$ and $f(2)$.

$f(-2) =$ ⬚

$f(2) =$ ⬚

(c) (4 points) Suppose that $g(x) = x^2 - 4$. Find $g(f(-2))$ and $f(g(4))$. Be sure to **show all work** to receive credit.

$g(f(-2)) =$ ⬚

$f(g(4)) =$ ⬚